14

QUESTION PAPER SERIES CODE
A

Centre Name : ______________________

Roll No. : ______________________

Name of Candidate : ______________________

S A U

Entrance Test for M.Phil./Ph.D. (Applied Mathematics), 2014

[PROGRAMME CODE : PAM]

Time : 3 hours *Maximum Marks* : 70

INSTRUCTIONS FOR CANDIDATES

Candidates must carefully read the following instructions before attempting the Question Paper :

(i) Write your Name, Roll Number and Centre Name in the space provided for the purpose on the top of this Question Paper and in the OMR/Answer Sheet.

(ii) This Question Paper has Two Parts : Part—A and Part—B.

(iii) Part—A (Objective-type) has 30 questions of **1** mark each. All questions are compulsory.

(iv) Part—B (Objective-type) has 40 questions of **1** mark each. All questions are compulsory.

(v) Symbols have their usual meanings.

(vi) **Please darken the appropriate Circle of 'Question Paper Series Code' and 'Programme Code' on the OMR/Answer Sheet in the space provided.**

(vii) Part—A and Part—B (Multiple Choice) questions should be answered on the OMR/Answer Sheet.

(viii) Answers written by the candidates inside the Question Paper will **NOT** be evaluated.

(ix) Calculators and Log Tables may be used. Mobile Phones are **NOT** allowed.

(x) Pages at the end have been provided for Rough Work.

(xi) **Return the Question Paper and the OMR/Answer Sheet** to the Invigilator at the end of the Entrance Test.

(xii) **DO NOT FOLD THE OMR/ANSWER SHEET.**

INSTRUCTIONS FOR MARKING ANSWERS IN THE 'OMR SHEET'

Use BLUE/BLACK Ballpoint Pen Only

1. **Please ensure that you have darkened the appropriate Circle of 'Question Paper Series Code' and 'Programme Code' on the OMR Sheet in the space provided.**

   Example :

   Question Paper Series Code

   Write Question Paper Series Code A or B and darken appropriate circle.

   [ ] A or B

   ●

   Ⓑ

   Programme Code

   Write Programme Code out of 14 codes given and darken appropriate circle.

   Write Programme Code [ ]

MEC	○	MAM	○	PCS	○
MSO	○	MLS	○	PBT	○
MIR	○	PEC	○	PAM	●
MCS	○	PSO	○	PLS	○
MBT	○	PIR	○		

2. Use only Blue/Black Ballpoint Pen to darken the Circle. Do not use Pencil to darken the Circle for Final Answer.
3. Please darken the whole Circle. ●
4. Darken <u>ONLY ONE CIRCLE</u> for each question as shown below in the example :

   Example :

Wrong	Wrong	Wrong	Wrong	Correct
● ⓑ ⓒ ●	~~ⓐ~~ ⓑ ⓒ ⓓ	~~ⓐ~~ ⓑ ⓒ ~~ⓓ~~	⦿ ⓑ ⓒ ●	ⓐ ⓑ ⓒ ●

5. Once marked, no change in the answer is allowed.
6. Please do not make any stray marks on the OMR Sheet.
7. Please do not do any rough work on the OMR Sheet.
8. Mark your answer only in the appropriate circle against the number corresponding to the question.
9. There will be <u>no negative marking</u> in evaluation.
10. Write your six digits Roll Number in small boxes provided for the purpose; and also darken appropriate circle corresponding to respective digits of your Roll Number as shown in the example below.

    Example :

ROLL NUMBER

1	3	5	7	2	0
●	①	①	①	①	①
②	②	②	②	●	②
③	●	③	③	③	③
④	④	④	④	④	④
⑤	⑤	●	⑤	⑤	⑤
⑥	⑥	⑥	⑥	⑥	⑥
⑦	⑦	⑦	●	⑦	⑦
⑧	⑧	⑧	⑧	⑧	⑧
⑨	⑨	⑨	⑨	⑨	⑨
⓪	⓪	⓪	⓪	⓪	●

PART—A

1. Let $\{f_n\}$ be a sequence of functions where $f_n : [0, 1] \to R$ is defined by $f_n(x) = x^n$ for $n = 1, 2, \ldots$. Then the sequence $\{f_n\}$ is

   (a) convergent but not uniformly convergent on $[0, 1]$

   (b) uniformly convergent on $[0, 1]$

   (c) not convergent on $[0, 1]$

   (d) convergent to a continuous function on $[0, 1]$

2. Let $a_1, a_2, \ldots, a_n, \ldots$ be a sequence of positive real numbers such that

$$\sum_{n=1}^{\infty} a_n^2$$

converges. Which of the following necessarily holds?

   (a) $\sum_{n=1}^{\infty} a_n$ converges

   (b) $\sum_{n=1}^{\infty} \frac{a_n}{n}$ converges

   (c) $\sum_{n=1}^{\infty} n\, a_n$ converges

   (d) None of the above

3. Let f be a real valued continuous function defined on $[0, 1]$ such that

$$\int_0^1 f(x)\,dx = 0$$

Then

   (a) $f \equiv 0$

   (b) $\int_0^x f(t)dt = 0$ for all x

   (c) there is a subinterval of $[0, 1]$ on which f is non-negative

   (d) there is always a proper subinterval $[\alpha, \beta] \subset [0, 1]$ on which $f(x) = 0$

4. Let $f(x) = e^{-x^2}$ and $g(x) = \frac{1}{1+x^2}$. Which of the following is true?

(a) $f(x) \geq g(x)$ for all $x \geq 0$

(b) $f(x) \leq g(x)$ for all $x \geq 0$

(c) $f(x) - g(x)$ changes sign finitely many times as x varies over $[0, \infty)$

(d) $f(x) - g(x)$ changes sign infinitely many times as x varies over $[0, \infty)$

5. The improper integral

$$\int_0^1 x^{m-1}(1-x)^{n-1}dx$$

exists if and only if

(a) $m > 0, n > 0$

(b) $m > 0, n < 0$

(c) $m < 0, n > 0$

(d) $m < 0, n < 0$

6. Let f be an increasing function and g be a decreasing function on an interval such that $f \circ g$ exists. Then $f \circ g$ is

(a) increasing

(b) decreasing

(c) monotone

(d) neither increasing nor decreasing

7. A group with at least two elements but with no proper non-trivial subgroups must be

(a) finite and of odd order

(b) finite and of prime order

(c) finite and of even order

(d) finite with no restriction over the order

8. $\mathbb{Z}_3 \times \mathbb{Z}_4$ is of order

(a) 3

(b) 4

(c) 12

(d) None of the above

9. Let G be an Abelian group of order 72. Number of subgroups of G of order 4 is

(a) 1

(b) 9

(c) 4

(d) Not a fixed number

10. If H and N are subgroups of a group G, and N is normal in G, then $H \cap N$ is always normal in

(a) G

(b) H

(c) N

(d) Not a normal group

11. A group is simple if

(a) it is non-trivial and has no proper non-trivial normal subgroups

(b) it is non-trivial and has proper non-trivial normal subgroups

(c) it is non-trivial and has no proper non-trivial subgroups

(d) it is non-trivial and has proper non-trivial subgroups

12. Which of the following is true?

(a) Every field is also a ring

(b) Every ring has a multiplicative identity

(c) Every ring with unity has at least two units

(d) Every ring with unity has at most two units

13. The order of the matrix ring $M_2(Z_2)$ is

(a) 4

(b) 2

(c) 16

(d) 8

14. The necessary and sufficient condition for the differential equation $f(x, y)dx + g(x, y)dy = 0$ to be exact is given by

(a) $\frac{\partial f}{\partial y} = \frac{\partial g}{\partial x}$

(b) $\frac{\partial f}{\partial x} = \frac{\partial g}{\partial y}$

(c) $\frac{\partial f}{\partial x} = \frac{\partial g}{\partial x}$

(d) $\frac{\partial f}{\partial y} = \frac{\partial g}{\partial y}$

15. The necessary condition for the partial differential equation

$$A\frac{\partial^2 u}{\partial x^2} + B\frac{\partial^2 u}{\partial x \partial y} + C\frac{\partial^2 u}{\partial y^2} = 0$$

where $u = u(x, y)$ and A, B, C are functions of x and y, to be elliptic is given by

(a) $B^2 - AC = 0$

(b) $B^2 - 4AC > 0$

(c) $B^2 - 4AC = 0$

(d) $B^2 - 4AC < 0$

16. The bisection method of finding roots of non-linear equation falls under the category of —— methods.

(a) bracketing

(b) open

(c) random

(d) graphical

17. In —— method, a system is reduced to an equivalent diagonal form using elementary transformations.

(a) Jacobi

(b) Gauss elimination

(c) Gauss-Jordan

(d) Gauss-Seidel

18. The highest order of polynomial integrand for which Simpson's 1/3 rule of integration is exact, is

(a) first

(b) second

(c) third

(d) fourth

19. Given $n+1$ data pairs, a unique polynomial of degree —— passes through the $n+1$ data points.

(a) $n+1$

(b) n

(c) n or less

(d) $n+1$ or less

20. The truncation error in quadratic interpolation in an equidistant table is bounded by

(a) $\frac{h^2}{9\sqrt{3}} \max |f'''(\xi)|$

(b) $\frac{h^2}{\sqrt{3}} \max |f'''(\xi)|$

(c) $\frac{h^2}{9} \max |f'''(\xi)|$

(d) $\frac{h^2}{\sqrt{2}} \max |f'''(\xi)|$

21. How will the mass of the object affect the way it speeds up or slows down?

(a) No effect

(b) More massive objects are easier to accelerate and harder to decelerate

(c) More massive objects are harder to accelerate and harder to decelerate

(d) More massive objects are easier to accelerate and easier to decelerate

22. When an object is moving faster through a fluid, then

(a) the force of friction is greater

(b) the force of friction is less

(c) the force of friction is unaffected

(d) None of the above

23. The amount by which an objective function coefficient can change before a different set of values for the decision variables becomes optimal is the

(a) optimal solution

(b) dual solution

(c) range of optimality

(d) range of feasibility

24. A basic solution is called degenerate if

(a) the value of at least one of the basic variables is non-zero

(b) the value of all the basic variables is zero

(c) the value of at least one of the basic variables is zero

(d) the value of all the basic variables is non-zero

25. If in a simplex table, the relative cost $z_j - c_j$ is zero for a non-basic variable, then there exists an alternate optimal solution, provided

(a) it is a starting simplex table

(b) it is an optimal simplex table

(c) it can be any simplex table

(d) None of the above

26. Let S be a non-empty closed convex set. Then

(a) S has finite number of vertices

(b) S has infinite number of vertices

(c) S may have finite or infinite number of vertices

(d) None of the above

27. The constraints of an LPP including non-negativity restrictions are

(a) either closed half spaces or hyperplanes

(b) closed half spaces

(c) hyperplanes only

(d) None of the above

28. Let X is any non-negative integer-valued random variable with its generating function

$$P(z) = \sum_{j=0}^{\infty} z^j P\{X = j\}$$

Now if it is given that X is Poisson with mean λ, then $P\{X \text{ is even}\}$ is given by

(a) $\dfrac{1+e^{-2\lambda}}{2}$

(b) $\dfrac{1+e^{-\lambda}}{2}$

(c) $\dfrac{1-e^{-2\lambda}}{2}$

(d) $\dfrac{1-e^{-\lambda}}{2}$

29. In an election, candidate A receives n votes and candidate B receives m votes, where $n > m$. Assuming that all orderings are equally likely, the probability that A is always ahead in the count of votes is

(a) $\dfrac{n-m}{2^{mn}}$

(b) $\dfrac{n-m}{2^{m+n}}$

(c) $\dfrac{2^{n-m}}{2^{m+n}}$

(d) $\dfrac{n-m}{n+m}$

30. The continuous random variable X is uniformly distributed with mean 1 and variance 3. Then $P(X < 0)$ is

(a) $\dfrac{1}{6}$

(b) $\dfrac{1}{2}$

(c) $\dfrac{1}{3}$

(d) $\dfrac{1}{4}$

PART—B

31. The matrix

$$A = \begin{pmatrix} 1 & 0 \\ 2 & 4 \end{pmatrix}$$

is given. Then eigenvalues of $4A^{-1} + 3A + 2I$ are

(a) 6, 15

(b) 9, 12

(c) 9, 15

(d) 7, 15

32. Which of the following system of vectors is linearly independent?

(a) $X_1 = (1, -1, 1)$, $X_2 = (2, 1, 1)$, $X_3 = (3, 0, 2)$

(b) $X_1 = (3, 1, -4)$, $X_2 = (2, 2, -3)$, $X_3 = (0, -4, 1)$

(c) $X_1 = (1, 6, 4)$, $X_2 = (0, 2, 3)$, $X_3 = (0, 1, 2)$

(d) None of the above

33. Among the following given statements which one is true?

(a) Any plane w in R^3 is a subspace of R^3

(b) Set of all real valued discontinuous function forms a subspace of V (Vector space of real valued function with real domain)

(c) Set of all continuous real valued functions f defined on the interval $[0, 1]$ forms a subspace of V (Vector space of real valued function with real domain)

(d) All of the above

34. If S_1 and S_2 be any two subsets of the vector space V, then which of the following statements is not correct?

(a) Span (S_1) = Span (S_2) if and only if $S_1 \subset$ Span (S_2) or $S_2 \subset$ Span (S_1)

(b) Span $(S_1 \cap S_2) \subseteq$ Span $(S_1) \cap$ Span (S_2)

(c) Span $(S_1) \cup$ Span $(S_2) \subseteq$ Span $(S_1 \cup S_2)$

(d) $S_1 \subseteq S_2$, then Span $(S_1) \cup$ Span (S_2) = Span $(S_1 \cup S_2)$

35. Let $L : R^3 \to R^3$ be a rotation about z-axis through an angle of $\frac{\pi}{3}$. Then its matrix A with respect to standard basis

(a) is diagonalizable

(b) is not diagonalizable

(c) A has eigenvalue 1 with algebraic multiplicity 2

(d) A has eigenvalue 2 with algebraic multiplicity 1

36. For a complex variable z, the function $f(z) = |z|^2$ is

(a) differentiable nowhere

(b) differentiable everywhere

(c) differentiable only at $z = 0$

(d) differentiable everywhere except at $z = 0$

37. Let C be the circle centered at origin and with radius 1. Then the value of the integral $\int_C \frac{e^z}{z^3} dz$ is

(a) $e^{\pi i}$

(b) πi

(c) $-\pi i$

(d) $e^{-\pi i}$

38. Under the stereographic projection, the point $z = \infty$ of the extended complex plane has the following corresponding point on the unit sphere.

(a) (0, 0, 0)

(b) (0, 0, 1)

(c) (0, 1, 0)

(d) (1, 0, 0)

39. For the function $f(z) = \frac{z - \sin z}{z^3}$, the point $z = 0$ is

(a) a removable singularity

(b) a pole

(c) an essential singularity

(d) None of the above

40. If C is the circle $|z| = \frac{1}{2}$, then $\int_C \frac{z^2 - z + 1}{z - 1} dz$ equals

(a) 1

(b) −1

(c) 0

(d) $\frac{1}{2}$

41. Which of the following statements is true?

(a) Cauchy-Riemann equations are necessary conditions for a function to be differentiable

(b) Cauchy-Riemann equations are sufficient conditions for a function to be differentiable

(c) Cauchy-Riemann equations are both necessary as well as sufficient conditions for a function to be differentiable

(d) Cauchy-Riemann equations are neither necessary nor sufficient conditions for a function to be differentiable

42. Let $f : [0, 1] \to R$ be a function such that $|f|$ is Riemann integrable over [0, 1]. Which of the following is necessarily true?

(a) f is Riemann integrable over [0, 1]

(b) f is Lebesgue integrable over [0, 1]

(c) f is both Riemann as well as Lebesgue integrable over [0, 1]

(d) f is neither Riemann nor Lebesgue integrable over [0, 1]

43. In an incomplete metric space

(a) no Cauchy sequence is convergent

(b) no convergent sequence has a convergent subsequence

(c) there is a Cauchy sequence which has no convergent subsequence

(d) there is a non-convergent sequence which has a convergent subsequence

44. Number of fixed points of the mapping $T: (0, 1) \to (0, 1)$ defined by $Tx = \frac{1}{x}$ is

(a) 0

(b) 1

(c) 2

(d) infinite

45. Which of the following statements is false?

(a) Every inner product space defines a norm

(b) Every inner product space defines a metric

(c) Every norm defines a metric

(d) Every metric defines an inner product space

46. The dual space of ℓ^3 is

(a) $\ell^{3/2}$

(b) $\ell^{2/3}$

(c) $\ell^{1/3}$

(d) $\ell^{1/2}$

47. Let H be a Hilbert space and $T : H \to H$ be a bijective bounded linear operator such that $T^* = T^{-1}$. Then

(a) T is self-adjoint

(b) T is unitary

(c) T is normal

(d) None of the above

48. Which of the following statements is false?

(a) Every countable set is measurable

(b) Cantor set is uncountable

(c) Cantor set is measurable

(d) Every measurable set is countable

49. If R is a ring with unity and N is an ideal of R containing a unit, then

(a) $N \subset R$

(b) $N \supset R$

(c) $N = R$

(d) None of the above

50. A Sylow 3-subgroup of a group of order 12 has order

(a) 4

(b) 9

(c) 12

(d) 3

51. Let p be a prime. A p-group is a group with the property that

(a) every element has order p

(b) at least one element has order p

(c) no element has order p

(d) one and only one element has order p

52. If $f(x) = x + 1$ and $g(x) = x + 1$, then in $\mathbb{Z}_2[x]$, $f(x) + g(x) =$

(a) $2x + 2$

(b) $2x$

(c) 2

(d) 0

53. Let E be the finite extension of degree n over a finite field F. If F has q elements, then E has

(a) q^n elements

(b) q elements

(c) nq elements

(d) Cannot say

54. Which of the following can be the order of a finite field?

(a) 4096

(b) 3127

(c) 36

(d) 60

55. The eigenvalues of the Sturm-Liouville problems are

(a) imaginary

(b) real

(c) both real and imaginary

(d) not defined

56. The solution of the total differential equation $xdy - ydx - 2x^2zdz = 0$ is given by

(a) $\frac{y}{x} - z^2 = c$

(b) $\frac{y}{x} + z^2 = c$

(c) $\frac{y}{x} - z = c$

(d) $\frac{y}{x} + z = c$

57. If $J_n(x)$ defines the Bessel's function of the first kind and of order n, then which of the following is true for $n > 2$?

(a) $\frac{d}{dx}(x^n J_n(x)) = x^n$

(b) $\frac{d}{dx}(x^n J_n(x)) = J_{n-1}(x)$

(c) $\frac{d}{dx}(x^n J_n(x)) = x^n J_{n-1}(x)$

(d) $\frac{d}{dx}(x^n J_n(x)) = 0$

58. The generating function for the Legendre polynomial is given by

(a) $(1-2xz)^{-1/2}$

(b) $(1-z^2)^{-1/2}$

(c) $(1-2x+z)^{-1/2}$

(d) $(1-2xz+z^2)^{-1/2}$

59. The orthogonal trajectories of the system of curves

$$\left(\frac{dy}{dx}\right)^2 = \frac{a}{x}$$

is

(a) $9a(y+c)^2 = 4x^3$

(b) $9a(y+c)^2 = x$

(c) $9a(y+c)^2 = 2x^2$

(d) $9a(y+c) = 4$

60. If $y = e^{ax}$ is a solution of the linear second-order ordinary differential equation

$$\frac{d^2y}{dx^2} + p(x)\frac{dy}{dx} + q(x)y = 0$$

then which of the following holds true?

(a) $a^2 + p(x)a + q(x) = 0$

(b) $a^2 - p(x)a + q(x) = 0$

(c) $a^2 + p(x)a - q(x) = 0$

(d) $a^2 - p(x)a - q(x) = 0$

61. The solution of the initial value problem

$$\frac{d^2y}{dx^2} - 1 = 0,\ y(0) = 1,\ \left(\frac{dy}{dx}\right)_{x=0} = 2$$

is given by

(a) $y(x) = \frac{x^2}{2} - 2x + 1$

(b) $y(x) = \frac{x^2}{2} + 2x - 1$

(c) $y(x) = \frac{x^2}{2} + 2x + 1$

(d) $y(x) = \frac{x^4}{2} + 4x + 1$

62. The general solution of the partial differential equation

$$\frac{\partial^2 z}{\partial x \partial y} = x + y$$

is of the form

(a) $\frac{1}{2}xy(x + y) + F(x) + G(y)$

(b) $\frac{1}{2}xy(x - y) + F(x) + G(y)$

(c) $\frac{1}{2}xy(x - y) + F(x)G(y)$

(d) $\frac{1}{2}xy(x + y) + F(x)G(y)$

63. Which of the following equations is parabolic?

(a) $f_{xy} - f_x = 0$

(b) $f_{xx} + 2f_{xy} + f_{yy} = 0$

(c) $f_{xx} + 2f_{xy} + 4f_{yy} = 0$

(d) None of the above

64. When the given equation cannot be reduced to any of the standard form, then to solve the differential equation, we apply

(a) Lagrange's method

(b) Charpit's method

(c) Monge's method

(d) Picard's iteration method

65. The complete integral of the equation $p^2x + q^2y = z$ is

(a) $\sqrt{(1-a)z} = \sqrt{ax} - \sqrt{y} + b$

(b) $(1+a)z = ax + y + b$

(c) $\sqrt{(1+a)z} = \sqrt{ax} - \sqrt{y} + b$

(d) $\sqrt{(1+a)z} = \sqrt{ax} + \sqrt{y} + b$

66. Applying Charpit's method, solution of the equation $(p+q)(px+qy) = 1$ is

(a) $z + b = \frac{2}{\sqrt{1+c}}(cx + y)^{1/2}$

(b) $z + b = \frac{2}{\sqrt{1+c}}(cy + x)^{1/2}$

(c) $z + b = \frac{2}{\sqrt{1+c}}(cx + y)$

(d) $z + b = \frac{2}{\sqrt{1+c}}(cy + x)$

67. Which of the following is Lagrange's linear partial differential equation?

(a) $Rr + Ss + Tt = V$

(b) $\dfrac{dx}{P} = \dfrac{dy}{Q} = \dfrac{dz}{R}$

(c) $Pdx + Qdy = R$

(d) $Pp + Qq = R$

68. LU decomposition of a square matrix

(a) is unique

(b) is not unique

(c) does not exist

(d) may be unique or not unique

69. The principle of least action

(a) is a variational principle

(b) when applied to the action of a mechanical system, can be used to obtain equation of motion of the system

(c) being applied in the theory of relativity, quantum mechanics and quantum field theory

(d) All of the above

70. The Euler-Lagrange equation is

(a) $\dfrac{\partial L}{\partial q} - \dfrac{d}{dt}\left(\dfrac{\partial L}{\partial \dot{q}}\right) = 0$

(b) $\dfrac{\partial L}{\partial q} - \left(\dfrac{\partial L}{\partial \dot{q}}\right) = 0$

(c) $\dfrac{\partial L}{\partial q} - \dfrac{d}{dt}\left(\dfrac{\partial L}{\partial q}\right) = 0$

(d) $\dfrac{\partial L}{\partial q} - \dfrac{d}{dt}\left(\dfrac{\partial L}{\partial \dot{q}}\right) = 1$

★ ★ ★